Classical Mathematics

ONE

A Guidebook to High School Geometry Through Primary Sources

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Preface

This guidebook represents a marked departure from the traditional textbook method of teaching and studying mathematics. Instead of reducing classic treatises of mathematics into pre-digested bits for students to consume as most textbooks are wont to do, this book will help students to read and digest these classics for themselves. In fact, this guidebook is meant to help students create their own personalized textbook. The great masters of the past are our teachers, and this book will help students learn from them.

It is my prayer that this book will, despite its faults, inspire students with a love for mathematics rooted in a divine vision of its true purpose. Mathematics is one of God's silent heralds.

I can think of no better reason to study than to follow the various paths of knowledge God has laid out for us to discover. If we follow the paths diligently, ducking our heads low under branches, and occasionally crawling, we will discover they all lead to the same place, because the trails were all cut by the same gardener. Poetry and mathematics converge.

This book represents my attempt to help students make such a journey for themselves. That is the spirit in which I wrote it, and the spirit in which I hope it will be received—and may God protect all who read it from any errors I have introduced.

* * *

This book would not have been possible without my students who gave me the opportunity to teach and refine my curriculum. I also owe a great debt to my entire family for their encouragement, and especially to my wife, Haley, for supporting me and allowing me to work many long evenings to complete this first stage of the Classical Mathematics curriculum series.

Daniel Maycock
July 2017

How to Use This Book

Required Books

Because this is a guide and not a complete textbook in itself, this book will guide you through portions of the following 3 books. Make sure you have a copy of each.

- **Euclid's *Elements*** (I recommend the Green Lion Press edition. Whatever edition you get, make sure it contains at least Books I, II, III, V, and VI.)
- ***Introduction to Arithmetic* by Nicomachus** (For information on how to purchase a copy, please visit *www.polymathclassical.com*.)
- ***Mathematics for the Nonmathematician* by Morris Kline**

Journal

In addition to the books listed above, you will need a notebook in which to take notes and complete the exercises. I recommend investing in a high-quality journal with plain, unlined paper. At the end of your studies, you will have written and compiled your own textbook, so it's worth using quality materials and doing your work neatly.

If you struggle to write neatly on unlined paper, consider drawing your own lines, using a light pencil and ruler. Or you may even write on lined notebook paper that you then cut and paste into your journal. However you choose to do it, the point is this: Your notebook must be organized and neat. Your work should not to resemble the scribblings of a madman. Instead, take for your example the notebooks of Leonardo da Vinci. Look up images of his notebooks and model your own notebook (in terms of neatness and beauty) on his.

It is absolutely essential to your success that you do the work by hand, using pen (or pencil) and paper. Even if your handwriting is nearly illegible, you must write longhand to get the full benefit of this guide. Taking shortcuts now will delay you later. (The only exceptions to this rule are the essays. Those may be typed.)

Structure

Each Cycle represents a week's worth of work and is subdivided into four Epicycles. Ideally, you will complete one Epicycle per day and reserve the fifth day to view the weekly videos that accompany this book or discuss the week's work with fellow students.

You will find that not all Cycles and Epicycles are equally weighted. In other words, some days and weeks will require more time and effort than others. You will be tempted to grumble at heavier days and slack off during lighter ones. However, you must resist this. The days with less work are gifts. Use your extra time and energy to explore whatever interests you in our reading. Mathematics is a grand puzzle, so enjoy it. Unleash your curiosity.

CYCLE 1

Mathematics for the Nonmathematician - Morris Kline
Chapters 1 - 3

After completing this cycle, students should be able to answer the following questions.

- What characterized Egyptian and Babylonian mathematics?
- What role did the Greeks play in the history of mathematics?
- What does it mean to say that mathematics deals with abstractions?

Epicycle A

Introduction

As we embark on any study, whether of music, poetry, or mathematics, it is important to know some of the discipline's history. For instance, imagine how unremarkable Beethoven would seem to someone who knows nothing of Mozart and the classical period. Or imagine how much one would miss if one read Virgil's *Aeneid* without knowing about Homer's *Iliad*. The past illuminates the present and make us capable of better understanding and appreciating each part of the story along the way. Unfortunately, most people study math by jumping in at the end of the story, never knowing who discovered the great theorems and equations that they learn to use. Now, most people probably don't care where the equations were born, so long as they work. Individuals like that lack intellectual curiosity (and won't be making any discoveries). But I'm going to assume that you're a different sort. After all, haven't you ever wondered where plus signs and minus signs came from and why we are always searching for *x* instead of *t* or *w*? And haven't you ever wondered if math ever drove someone insane?[1] And haven't you ever wondered why everyone is forced to study math, even if they don't care two snaps about science or technology and all that stuff math is usually blamed for? Well, here's where you can begin to find out...

<u>Read</u>

- ☐ Chapter 1 in *Mathematics for the Nonmathematician*[2]

<u>Respond</u>

- ☐ In your journal, write 3 - 5 detailed paragraphs summarizing Chapter 1.

  Your writing should be clear and consist of complete sentences. Your paragraphs should also be informative. (In other words, no 2-sentence paragraphs. Make your summary worthwhile.) If you have trouble summarizing the whole chapter this way, identify several key paragraphs summarize those.

- ☐ In your journal write down your answers for the all three exercises from page 10.

  You may want to copy the original question into your journal along with your responses so that your answers will make sense when you review your journal entries later.

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[1] It has, and more than once. One mathematician (Wolfgang Bolyai) wrote to his son, who was also a mathematician, warning him not to try to solve a particular problem for fear it would drive him into a maddening despair.

[2] Make a habit of taking notes in your journal as you read. Note anything which seems confusing or interesting. Write down your thoughts, your questions, your observations, etc. This will also help you complete the assignments with ease.

Epicycle B

Introduction

As you may have suspected from yesterday's reading, Kline doesn't care much for Christianity and Church history. So, don't be surprised that in today's reading, Kline implies that Christianity—or at least the Church—is to blame for the loss of Greek knowledge. However, until the fall of the Roman Empire, Christians had been great students of Greek learning. In fact, the Emperor Julian the Apostate sought to ban Christians from teaching Greek literature because they were embarrassing Paganism by using its books to teach about Christ. For instance, a thoughtful Christian can learn much by carefully comparing Jesus and Achilleus. At any rate, this just goes to show that the loss of Greek knowledge in the Middle Ages had less to do with the attitude of the Church and more to do with the tremendous social and political upheavals caused by the collapse of the Roman Empire.

Nevertheless, Kline does a number of things well in Chapter 2. Pay attention to how many different people groups contributed to the development of mathematics, and notice what kind of advancements occur along the way. Many of these profound insights seem absurdly simple. However, making a true discovery, or coming up with an original idea is one of the most difficult things to do. The introduction of 0 as a number is an idea so simple that it wasn't thought up for centuries, yet was as powerful as the invention of gunpowder.

<u>Read</u>

- ☐ Chapter 2 in *Mathematics for the Nonmathematician*[3]

<u>Respond</u>

- ☐ In your journal, write 3 - 5 paragraphs summarizing passages from Chapter 2 that you found particularly interesting.

  Your writing should be clear and consist of complete sentences. Your paragraphs should also be informative. (In other words, no 2-sentence paragraphs. Make your summary worthwhile.) If you have trouble summarizing the whole chapter this way, identify several key paragraphs summarize those.

- ☐ In your journal, write down your answers for exercises 1 - 6 from page 28. Make sure to write legibly in complete sentences.

  You may want to copy the original question into your journal along with your responses so that your answers will make sense when you review your journal entries later.

[3] Don't forget to take notes in your journal as you read.

Epicycle C

Introduction

There's a reason why the Greeks were so attracted to mathematics. They reckoned that something that always exists is more real than something that exists for only a moment. Furthermore, they reasoned, if something is constantly changing, how do we know it's the same thing and not some new thing? For instance, imagine you have an old boat that's in constant need of repairs. Every year you replace the rotted planks with new ones, until one day, you've replaced every piece of the boat. Is that boat still the same boat it once was if none of the original parts remain? Or consider the difference between a frolicking lamb and the dull old sheep it becomes. A lamb is different than a grown sheep in many ways, so in what sense is a lamb the same thing as the old ewe it grows to be? These sorts of problems troubled the Greeks, and they reasoned that although physical things are constantly changing—growing old, wearing down, springing up, changing color and size and shape—we can still identify them because they *participate* in an unchanging form. In other words, we understand the form (or concept) of 'sheep,' and as long as the lamb or ewe fits into that concept, we recognize it as the same thing. Once the lamb becomes dinner, however, the concept no longer applies, so we can't call it a sheep because it no longer participates in the form of 'sheep.' The same is true of trees turning into tables. We recognize trees as trees—despite the fact that no two trees are identical—because they participate in the form of 'tree.' But once the tree becomes a table, a new form applies: table.

The Greek concept of form can be difficult to grasp, but the important thing is this: the Greeks believed that ultimate knowledge of the truth could not rely upon physical things, because the physical world is in a state of constant change. Thus, the only things that could be known for sure were unchanging truths, truths that do not depend upon the physical world, truths that are found in the invisible world of pure reason—like the abstract, invisible, immaterial world of mathematics.

<u>Read</u>

- ☐ Chapter 3 in *Mathematics for the Nonmathematician* (You may skip sections 3-4 "Methods of Reasoning" & 3-5 "Mathematical Proof.")

<u>Respond</u>

- ☐ In your journal, write 3 - 5 paragraphs summarizing the important points of sections 3-1 & 3-2.

  Remember that you may later use these paragraphs for review, so make sure to be detailed and precise in your summaries now to save yourself time and effort later.

Epicycle D

Introduction

You may be surprised to see that today's work includes an essay assignment. You'll write several essays as part of this course, and perhaps it's worth explaining why.

Have you ever tried to tell someone about something, only to find halfway through your explanation that you really didn't know what you were talking about? As it turns out, one of the best ways to make sure you've understood something is to explain it to someone else. If you can't explain an idea clearly, you clearly haven't understood it. So, one reason to write essays is to give yourself the opportunity to test your knowledge and fill in the gaps.

A second reason to write essays is that clear and precise thought requires clear and precise language. Thinking well will come more easily to you the more you practice careful writing.

A third reason to write essays is that mathematicians are expected to be able to write clearly and eloquently about mathematics. Mathematics is a discipline of beauty, and mathematicians appreciate elegance in an essay as well as in a mathematical proof.

Read

- ☐ Review Chapter 3. Reread any passages that you found confusing or interesting yesterday.

Respond

- ☐ In your journal, write your answers to the exercises on page 39 and page 51.

  Be sure to thoughtfully consider the questions before attempting to answer them. Make sure to write legibly in complete sentences.

Essay

Write a 500-800 word essay summarizing the development of mathematics as presented by Chapter 2. Make sure to focus on the contributions of the Greeks.[4]

- ☐ You don't need to finish the essay today, but you do need to produce a detailed outline so that you can finish the essay over the next few days. (This is where the summaries you wrote on Chapter 2 may come in handy.)

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[4] Here and elsewhere, a formal presentation on the topic may be substituted for the essay.

CYCLE 2

Mathematics for the Nonmathematician - Morris Kline
Chapter 4

Introduction to Arithmetic - Nicomachus
Book I, Chapters 1 - 8

After completing this cycle, students should be able to answer the following questions.

- What is number?
- What are the four "scientific methods" of mathematics?
- With what are each of the four methods concerned?
- What are the properties of the even-times even numbers?

Epicycle A

Introduction

Although we will spend most of the year doing geometry, it's worth taking some time to explore arithmetic first. As you'll see, arithmetic is logically prior to geometry; that is, the structure of geometry is built on arithmetic. Of course, you've already had several years of arithmetic, but there's more to arithmetic than learning to add, subtract, multiply, and divide numbers.

Consider, for instance, the concept of number. When we discuss numbers, we aren't talking about physical things. Mathematically, seven is not 7 books or 7 dogs or 7 of anything. It's just sevenness by itself. You may be tempted to think that the numeral 7 is seven. But it isn't. It would be false to say that the numeral sevens on this page are real sevens. After all, they're just blots of ink on paper. Numbers are concepts, and concepts, like all ideas, are invisible and immaterial and exist only in the mind. This doesn't mean, however, that numbers or ideas are less real than your chair. But it does mean that they exist in a different way.

The numerals we write, just like words, carry a meaning. The sevens I wrote earlier *mean* 7, but they can't *be* 7. Similarly, the words on this page aren't really *on* the page; they're *represented* on the page. The blots of ink you read as words are just blots of ink. The word you see is the form and the meaning the ink depicts. It's much like a painting. The people in a painting aren't really *in* the painting—and good thing, too. Otherwise, portraiture would probably be illegal.

<u>Read</u>

- ☐ Chapter 4, sections 4-1 and section 4-2 in *Mathematics for the Nonmathematician* [5]

<u>Respond</u>

- ☐ In your journal, write 3 - 5 paragraphs summarizing the important points of sections 4-1 & 4-2.

  Remember that you will later use these paragraphs for review, so make sure to be detailed and precise in your summaries now to save yourself time and effort later.

Essay

- ☐ Revise, edit, and complete the outline for your essay.

  Remember that a detailed and well-organized outline will make your essay much easier to write.

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[5] Don't forget to journal your brilliant observations, etc. as you read.

Epicycle B

Introduction

Today we begin reading Nicomachus of Gerasa, a Greek mathematician who lived around A.D. 100, which places him about 400 years after Euclid. Although we know little about his life, it's clear from his writings that he was a Pythagorean. Nicomachus wrote several books of mathematics, but his most famous is *Introduction to Arithmetic*.

Introduction to Arithmetic is exactly what it sounds like. It's a book about basic arithmetic, although some of what Nicomachus considers basic arithmetic may not be what you expect. It's all very impressive stuff, but we can't give Nicomachus all the credit. In the ancient world, it was common to borrow insights from earlier mathematicians and include them along with your own discoveries. Thus, much of what Nicomachus includes was already well known. But that shouldn't really be a surprise. After all, if his book contained only new ideas, it could hardly be called an introduction.

There are several things I love about Nicomachus. First, he enjoys numbers more than any other person I've encountered. Second, he presents astonishing relationships within numbers and number sets. Third, because he's a Pythagorean, he says all sorts of bizarre things you would never read in a modern textbook. Enjoy it!

<u>Read</u>

- ☐ Chapters 1 - 3 in *Introduction to Arithmetic*

<u>Respond</u>

In your journal, answer the following study questions for each chapter.[6]

- ☐ *Chapter 1*
  What is 'wisdom' according to this chapter?
  With what is wisdom particularly concerned?
- ☐ *Chapter 2*
  What is the difference between magnitudes and multitudes?
  What does Nicomachus say wisdom is to be considered?
- ☐ *Chapter 3*
  With what is arithmetic concerned? What about music, geometry, and astronomy?
  According to this chapter, why is mathematics necessary and for what is it useful?

Essay

- ☐ Begin writing your essay from your outline. You will finish this rough draft tomorrow, so all you need to do today is write the first half.

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[6] Remember, it's a good idea to copy the original question into your journal along with your responses so that your answers will make sense when you review your journal entries later.

Epicycle C

Introduction

So far, *Introduction to Arithmetic* is more a book of philosophy than a book of mathematics. The reason for this is that Nicomachus is arguing for the importance and superiority of mathematics. In today's reading, he'll go one step further and argue that among the mathematical disciplines, arithmetic should be considered first. These kinds of arguments are, by nature, philosophical ones. Mathematics cannot defend itself. A math equation alone cannot prove the importance or usefulness of math. The same is true of most disciplines. For instance, any time a scientist argues that science is good or useful or important, he isn't doing science. Whether he realizes it or not, he's doing philosophy.

If you remember yesterday's reading, Nicomachus even goes so far as to argue that mathematics and philosophy are closely related. While mathematics can be defended only by philosophy, Nicomachus suggests that to be a good philosopher, one must first study mathematics—because in mathematics one learns to think precisely and not be swayed by mere appearances. The obvious answer to a math problem is not always the correct one.

There's one more thing worth noting here. Pay special attention to Nicomachus' assumptions about the origin of mathematics. He doesn't argue that mathematics is an invented language used to *describe* natural phenomena. Instead, he says that mathematics *arises* from nature itself—an important distinction—because it was placed there by the "world-creating God." Thus, according to Nicomachus, mathematics is nothing less than a behind-the-scenes glimpse of the universe.

Read

- ☐ Chapters 4 - 6 in *Introduction to Arithmetic*

Respond

In your journal, answer the following study questions for each chapter.

- ☐ *Chapters 4 & 5*
  Why should arithmetic be studied first?
- ☐ *Chapter 6*
  Of what does "scientific number" consist?

Complete the following exercises.[7]

- ☐ Calculate *a - f* of Set 1 of the review exercises on page 91 in *Mathematics for the Nonmathematician.*

Essay

- ☐ Finish the rough draft of your essay.

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[7] For these kinds of drills, feel free to use scratch paper instead of recording them in your journal, although you're free to put them there, too.

Epicycle D

Introduction

In Chapter 7 we finally get to something that begins to look more like arithmetic, and, consequently, Nicomachus' vocabulary becomes more technical. A few explanations are in order.

First, remember that 0 is not a number to Nicomachus. He wasn't aware of the concept.

Second, he uses the terms "monad" and "dyad" at various points. The "monad" is merely that which exists as an indivisible unity—in other words, the number 1, which can't be easily divided into halves (like 2) or thirds (like 3) etc. He also occasionally calls the number 1 "unity," which makes sense. Similarly, the "dyad" is that quantity that exists as twoness—in other words, it's the number 2.

It's uncertain whether Nicomachus recognized 1 and 2 as numbers. At times he seems to assume that they are *parts* of number—that which numbers are made of—and at other times he seems to treat them as actual numbers. On the whole, however, the Greeks treated 2 as the first number. While 1 was simply "unity," they thought of number as "plurality." Thus, the first plural is 2, because it's the first number that can be divided simply.

If this seems strange, think of it this way. If you have one bean, you don't have a number of beans, you have *a bean.* If you have two beans, however, now you've got a number of beans. You can count two or more beans, but to the Greeks, it would be silly to count your beans if you've only got one.

Read

- ☐ Chapters 7 - 8 in *Introduction to Arithmetic*

Respond

In your journal, answer the following study questions for each chapter.

- ☐ *Chapter 7*
  What are the differences between odd and even numbers?
- ☐ *Chapter 8*
  What are the three divisions of the class of even numbers?
  What is the process for producing the even-times even numbers?

Complete the following exercises.

- ☐ Calculate *a, b, e, f, i, j, m, n* of Set 2 of the review exercises on page 91 in *Mathematics for the Nonmathematician.*

Essay

- ☐ Take time today and tomorrow to revise and edit your essay to produce a final draft.

CYCLE 3

Introduction to Arithmetic - Nicomachus
Book I, Chapters 9 - 16

After completing this cycle, students should be able to answer the following questions.

- What are the three divisions of the even, and what are their properties?
- What are the three divisions of the odd, and what are their properties?
- What does it mean for a number to be perfect or deficient or superabundant?

Epicycle A

Introduction

One of the things that begins to happen as we read Nicomachus is that numbers become individuals. Before learning that 64 is an even-times even number, you may have been tempted to assume that there was nothing unique about 64 and that it wasn't much different from 63 or 62. The truth is, however, that each number has its own personality.

Today we'll encounter two more divisions of even numbers: even-times odd and odd-times even. The names can be a bit confusing, so here's a way to keep from getting too puzzled.

The even-times even numbers arise from multiplying 2 times itself over and over again. Thus, we can think of the "even-times even numbers" as the "two-times two numbers."

The even-times odd numbers arise from multiplying 2 times any odd number. Thus, we can think of the "even-times odd numbers" as the "two-times odd numbers."

The odd-times even numbers arise from multiplying any odd number times any even number greater than 2. Thus, we can think of any of these numbers as "any odd number times any greater-than-two even number."

The names of these divisions of the even numbers are unwieldy, but once you grasp that the "even-times" in the even-times even and even-times odd numbers means 2-times, it gets a little easier keep them straight.

<u>Read</u>

- ☐ Chapters 9 - 10 in *Introduction to Arithmetic*

<u>Respond</u>

In your journal, answer the following study questions for each chapter.

- ☐ *Chapter 9*

  What are the properties of even-times odd numbers? (Esp. see paragraphs 1 & 2)

  What is the process for producing the even-times odd numbers?

  What is the relationship in the series of even-times odd numbers that Nicomachus is highlighting in paragraph 6?

- ☐ *Chapter 10*

  How are odd-times even numbers different from even-times odd numbers?

  What is the process for producing the odd-times even numbers?

  In what way does the produced series of odd-times even numbers have both the properties of the even-times even and the even-times odd?

This week, you will memorize the divisions of the even and odd. Copy the lists below into your journal. It will be a handy cheat sheet for tomorrow.

Even-times even: 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096
Even-times odd: 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, 46, 50
Odd-times even: 12, 20, 24, 28, 36, 40, 44, 48, 52, 56, 60, 68

Epicycle B

Introduction

While the even number series can be divided into three species of even numbers, the odd number series can be divided into only two: prime and composite. Although Nicomachus doesn't spend too much time on prime numbers, they are fascinating and hugely important. Until about the 1970s, prime numbers were thought to be mathematically interesting, but practically useless.

Today, however, prime numbers are used in cryptography, and encrypted connections over the internet would be impossible without them. In one sense, given how much we depend upon encryption for everything from secure email to online shopping and banking, prime numbers now run the world.

<u>Read</u>

- ☐ Chapters 11 - 12 in *Introduction to Arithmetic*

<u>Respond</u>

In your journal, answer the following study questions for each chapter.

- ☐ *Chapters 11 & 12*
  What is the first division of odd numbers, and what is its primary property?
  What is the second division of odd numbers, and what is its primary property?

In your journal, write out numbers 1 through 50 in a neat column.[8]

- ☐ Find and label the even-times even numbers
- ☐ Find and label the even-times odd numbers
- ☐ Find and label the odd-times even numbers
- ☐ Find and label the prime numbers
- ☐ Find and label the composite odd numbers

(If you have time, get a head start on tomorrow's work by writing out numbers 51 through 100 and starting to label those.)

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[8] To save time, I recommend adopting shorthand labels for the names—for instance, ExE for even-times even and ExO for even times odd, etc. Or you might want to provide a color-coding key and highlight the numbers in different colors to categorize them. Whatever you do, just make sure your labels are clear and distinguishable.

Epicycle C

Introduction

So far, Nicomachus has divided the even into 3 species and the odd into 2. However, he now adds an additional category in Chapter 13 to match the 3 divisions of the even with 3 divisions of the odd. The division he makes here is that of the *relatively prime*. These are pairs of numbers that share no common factors, like 9 and 25.[9] But even with these 6 divisions, Nicomachus isn't done. In Chapter 14, he suddenly starts over and divides numbers into 3 new categories based on an entirely different principle: the sum of the factors.

There's a reason we're spending time on these divisions and categories. To understand the whole of something, one must understand the parts and the relationships between them. For instance, to completely understand a car engine, you must understand each bolt and spring and the role of each. It does no good to be able to identify pistons and o-rings and nuts if you don't know how they all fit and work together. Similarly, by carefully organizing numbers into different categories according to different principles, we learn to see how the parts fit together to make a wonderful and complex whole. The whole number series may have originally struck you as a bit monotonous, but we've now discovered that there's a rainforest of diversity hiding in this infinite procession of numbers.

<u>Read</u>

- ☐ Chapters 13 - 14 in *Introduction to Arithmetic*

<u>Respond</u>

In your journal, answer the following study questions for each chapter.

- ☐ *Chapter 13*
  - What is the third division of odd numbers, and what is its primary property?
  - What is the "sieve" of Eratosthenes?
  - What is the process for determining whether numbers are relatively prime with respect to each other?
- ☐ *Chapter 14*
  - What does it mean for a number to be superabundant?

In your journal, write out numbers 51 through 100 in a neat column.

- ☐ Find and label the even-times even numbers
- ☐ Find and label the even-times odd numbers
- ☐ Find and label the odd-times even numbers
- ☐ Find and label the prime numbers
- ☐ Find and label the composite odd numbers

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[9] Euclid also made this distinction many years before Nicomachus, but didn't limit the relatively prime to odd numbers (it's common to have odd and even number pairs that have no common factors, like 15 and 16).

Epicycle D

Introduction

Perfect numbers are extremely rare. Nicomachus lists only 4 because those were the only ones that had been discovered.[10] Even as late as 1900, most mathematicians were aware of only 11 perfect numbers. Since the invention of the computer, we've been able to find quite a few more. In 2016, the 49th perfect number was discovered. No one knows how many perfect numbers there are. Some speculate that there might be an infinite number, but we don't know for sure.

In case you were wondering, the 5th, 6th, and 7th perfect numbers are 33,550,336; 8,589,869,056; and 137,438,691,328. The 8th perfect number is 2,305,843,008,139,952,128. I won't even show you the 9th perfect number because it's 37 digits long. And before you ask, the 49th perfect number has nearly 45 million digits. Imagine having to write that out. That explains why most of our discoveries involving perfect numbers came only after the invention of computers. There's no other way we could make discoveries about million-digit numbers. After all, it took humanity thousands of years to discover the first 11 perfect numbers, and the 11th has a mere 65 digits.

<u>Read</u>

- ☐ Chapters 15 - 16 in *Introduction to Arithmetic*

<u>Respond</u>

In your journal, answer the following study questions for each chapter.

- ☐ *Chapter 15*
  What does it mean for a number to be deficient?
- ☐ *Chapter 16*
  What does it mean for a number to be perfect?

Return to your column of numbers 1-100 in your journal.

- ☐ Label all the perfect numbers.
- ☐ Find and label all the deficient numbers between 25 & 50.[11]
- ☐ Find all label the superabundant numbers between 25 & 50.

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[10] Euclid is often credited with the discovery of the first 4 perfect numbers. It was he who first proposed the method for the discovery of perfect numbers. (See *Elements* IX.36.)

[11] Use scratch paper to work this out. Write out all the factors for each number and see what they add up to. Some numbers, of course, will be obvious. For instance, prime numbers are always deficient because they have only a factor of 1 apart from the number itself.

CYCLE 4

Introduction to Arithmetic - Nicomachus
Book I, Chapters 17 - 23

After completing this cycle, students should be able to answer the following questions.

- What are the 10 divisions of relative quantity?
- What is the difference between a superparticular and a superpartient? (Give examples of each.)
- What is the difference between a subsuperparticular and a subsuperpartient? (Give examples of each.)

Epicycle A

Introduction

Guess what. Nicomachus is still categorizing numbers according to various properties. This time, however, Nicomachus gives a systematic account of relative quantity rather than absolute quantity. In other words, he's now organizing numbers according to how they relate to each other as *ratios* of factors to products, and vice versa.

In Chapter 18, he discusses the multiple and submultiple. Multiples are simply numbers that you get when you multiply some number times another number. For instance, 22 and 33 and 121 are multiples of 11.

Submultiples are a bit trickier. Basically, submultiples are factors. The way Nicomachus thinks of submultiples is this: Just as 10 is the double of 5, and 15 is the triple of 5, we can divide a number in the same way. We can take a number's half, or its third, or its fourth part, etc. Here's an example: Let's divide 24 into its submultiples using fractions (or ratios). In 24/12, the submultiple 12 is the half part. In 24/8, the submultiple 8 is the third part. In 24/6, the submultiple 6 is the 4th part, and so on. Essentially then, multiples and submultiples are inverses of each other. If we're multiplying numbers, the product is the multiple, and the factors we multiplied to achieve the multiple are each submultiples.

Before I end this section, there's one more observation I need to make. If you were paying attention, you've noticed that, strictly speaking, Nicomachus isn't doing arithmetic any more. Remember how he defined arithmetic in Chapter 3. Arithmetic, he said, deals with absolute quantity. These chapters, however, deal with relative quantity, which means that we're no longer doing arithmetic. We're making music.[12]

Read

- ☐ Chapters 17 - 18 in *Introduction to Arithmetic*

Respond

In your journal, answer the following study questions for each chapter.

- ☐ *Chapters 17 & 18*
  What are the highest division of relative quality?
  What are the 5 species of the greater?
  What are the 5 species of the less?

Complete the following exercises.

- ☐ Calculate *g, h, i, j, k* of Set 1 of the review exercises on page 91 in *Mathematics for the Nonmathematician.*
- ☐ Calculate *c, d, g, h, k, l* of Set 2 of the review exercises on page 91 in *Mathematics for the Nonmathematician.*

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[12] If it seems strange to consider these chapters musical, consider how a harp or piano works. Different strings produce different pitches due to the ratios between string lengths. The same is true of note durations, which depend upon the ratio between a note's value and the meter. Time and pitch are both relative quantities.

Epicycle B

Introduction

It is important to keep in mind that Nicomachus is still exploring the species of the greater and lesser. Of the 5 listed in Chapter 18, Chapter 19 addresses the second species of the greater and of the lesser, the superparticular and the subsuperparticular.

The key to understanding the superparticular and subsuperparticular is that each arises from a ratio or fraction.[13] The superparticular is a ratio or fraction in which the first number (the numerator) contains the whole of the second (the denominator) plus one part besides. Thus, 4:3 (or 4/3) is a superparticular because 3 fits into 4 once and leaves 1/3 as a remainder, since 4/3 is equal to 1 + 1/3 (or 3/3 + 1/3).

The subsuperparticular describes the inverse relationship. For instance, 3:4 (or 3/4) is a subsuperparticular because the first number, 3, fits once into the second number, 4, and leaves a one-part remainder, 1/4.

Essentially, the superparticular is a fraction that is greater than 1 by a remainder of *one* part (such as 1/3 or 1/4, but never 2/5), and the subsuperparticular is a fraction that is less than 1 by a remainder of one part. Of course, there are different kinds of superparticulars and subsuperparticulars. The first kind of superparticular is the sesquialter, which has the value 3:2 (or 3/2). This includes ratios of 6:4, 9:6, 12:8, etc. since they all share the same fractional value of 3/2. As you might expect, the subsesquialter ratio is merely the inverse of the sesquialter: 2:3, 4:6, 6:9, 8:12, etc. Notice that superparticulars and subsuperparticulars contain consecutive numbers in their simplest fractional form (2/3, 3/4, 4/5 etc.).

You may be wondering why the ratio 2:1 isn't considered part of this series. Well, if we look at it as a fraction, 2/1 is simply equal to 2. Because the half measures the double perfectly, we never have anything left over, unlike 3:2 or 4:3, which always leave a remainder.

<u>Read</u>

- ☐ Chapter 19 in *Introduction to Arithmetic*

<u>Respond</u>

- ☐ In your journal, show that the claim made in paragraph 19 is true by performing the calculation with the heteromecic numbers for all 10 perfect squares on the diagram in paragraph 9.[14]
- ☐ Copy out the following 8 fractions and analyze them to determine whether they are superparticular or subsuperparticular or neither.

$$\frac{36}{48} \quad \frac{65}{52} \quad \frac{30}{15} \quad \frac{16}{12} \quad \frac{14}{16} \quad \frac{25}{36} \quad \frac{25}{30} \quad \frac{18}{12}$$

[13] A ratio is typically written using a colon to separate the values. For instance, a ratio of 2 to 3 is written 2:3. However, this ratio can be helpfully rewritten as a fraction by replacing the colon with a fraction bar. Thus, 2:3 becomes 2/3.

[14] The heteromecic numbers are those which stand on either side of the diagonal of square numbers. For example, the heteromecic numbers for 4 and 9 are 6 and 6. Similarly, the heteromecic numbers for squares 25 & 36 are 30 and 30.

Epicycle C

Introduction

We next turn our attention to the third species of the greater and of the lesser: the superpartient and subsuperpartient. These are much like the superparticular and subsuperparticular. The difference lies in the remainder. If the remainder in a superparticular is 1/3 or 1/4 or 1/5 etc., the remainder in a superpartient is greater than one part (2/3 or 3/4 etc.).

Thus, the ratios that make up the superpartient are ratios of 5:3, 7:4, 9:5, etc. Notice that each of these is greater than 1 by more than 1 part. For example, 5/3 surpasses 1 by 2/3. The superparticular, on the other hand, is always greater than 1 by a single part. For instance, 4/3 is greater than 1 by 1/3, and 5/4 surpasses 1 by 1/4.

We know by now that to get the sub- form we merely invert the fractions. Thus, the subsuperparticular is always one part shy of one (2/3 *needs 1/3*, 3/4 *needs 1/4*, etc.), and the subsuperpartient needs more than a single part to get to one (3/5 *needs 2/5*, 4/7 *needs 3/7*, etc.).[15]

The last two species of the greater and of the lesser are the multiple forms of the superparticular, subsuperparticular, superpartient, and subsuperpartient. To use the multiple superparticular as an example, it's a ratio or fraction with the same type of remainder as a regular superparticular. The difference is that the larger number contains the smaller more than once. Thus, 7/3 is a multiple superparticular because 7 contains 3 twice, but leaves a remainder of 1/3, just like the ordinary superparticular. Similarly, in a multiple superpartient like 15/4, the 15 contains 4 thrice and leaves a remainder of 3/4.

While all this may seem strange and needlessly difficult, the point to take away is this: ratios and fractions, just like whole numbers, may be categorized according to their properties. We might even go one step further and say that because ratios can be rewritten as fractions, they belong in arithmetic after all, since a fraction is just a clever way of writing a number that falls in the gaps left by the wholes.[16]

<u>Read</u>

- ☐ Chapters 20 - 22 in *Introduction to Arithmetic*

<u>Respond</u>

- ☐ Copy out the following 10 fractions and label them as you determine which of the 5 species of the greater (or 5 species of the lesser) each is.

$$\frac{27}{18} \quad \frac{8}{12} \quad \frac{25}{10} \quad \frac{5}{6} \quad \frac{60}{24} \quad \frac{32}{40} \quad \frac{28}{16} \quad \frac{8}{14} \quad \frac{6}{23} \quad \frac{14}{5}$$

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[15] Observe that Nicomachus' series does not account for all superpartient ratios. For example, 7/5 is also a superpartient because it exceeds 1 by more than 1 part.

[16] It's not the best pun I ever wrote, but it's probably the first time a joke has appeared on the same page as the word "subsuperpartient."

Epicycle D

Introduction

Today's reading brings Book I of *Introduction to Arithmetic* to a close. Over the course of 23 chapters, Nicomachus has classified numbers with the zeal of a botanist classifying rare species in an unexplored jungle. Thanks to these efforts, we now have quite a few categories for understanding and analyzing numbers.

Division based on Odd & Even	Division based on Factors	Division based on Relation
Odd	Superabundant	Multiple
Even	Deficient	Submultiple
Even-times even	Perfect	Superparticular
Even-times odd		Subsuperparticular
Odd-times even		Superpartient
Prime		Subsuperpartient
Composite odd		Multiple superparticular
Relatively prime		Multiple subsuperparticular
		Multiple superpartient
		Multiple subsuperpartient

As we've seen, Nicomachus takes time to divide and classify numbers so thoroughly because he values arithmetic on several levels. In today's reading, he even suggests that mathematics is related to virtue. Because vice is the absence of limit—a lack of self-control—a virtuous person limits his passions through reason and discipline. Similarly, mathematics imposes limits upon magnitude and multitude, which are otherwise infinite (or undefined) and therefore impossible to grasp.[17] We might conclude from this that the rigor of mathematical study trains the soul toward balance and virtue. Thus, we may hope that studying mathematics will make us better persons. It should, at very least, make us better philosophers.

As you read Chapter 23, notice how he again insists that the orderliness of number is something created by God. Mathematics isn't a language humans have invented. Instead, it's a language we use to communicate with the hidden structure of reality itself. In one sense, mathematics gives us direct access to the mind of God.

Read

☐ Chapter 23 in *Introduction to Arithmetic*

Essay

Write a 500 - 800 word essay explaining the importance and value of mathematics according to Nicomachus.

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[17] The sense in which the infinite is made finite in mathematics is not by making the infinite smaller somehow. Instead, it is made finite through definition. Think of it like this: something that isn't defined can't be studied.

CYCLE 5

The Elements, Book I - Euclid
Definitions - Common Notions

After completing this cycle, students should be able to answer the following questions.

- Why does Euclid begin with a set of definitions and axioms?
- What is the difference between a postulate and a common notion?
- What is the controversy surrounding the 5th postulate?

Epicycle A

Introduction

Today we begin reading Euclid's *Elements*, the book that will occupy most of our time during the year. Unfortunately, we don't know much about Euclid's life. In fact, we're only relatively sure about two facts: first, that he was a Greek mathematician living some time between 347B.C. and 287B.C, and second, that he worked in Alexandria. It's also likely that he studied mathematics at Athens from geometers that had studied under Plato. In any case, *The Elements* is a masterpiece of mathematics and is consistently ranked among the greatest books of all time. It may take a while to see why, but trust me—it's beautiful.

Read

- ☐ Definitions, postulates, and common notions of Book I[18]

Respond

Consider the following questions carefully. In your journal, write your answer to each question legibly and in full sentences. Your answers should be thoughtful. Hasty answers provided without careful thought are wastes of time and ink. Also, know that if you find it particularly difficult to answer a question, it is acceptable to write down your own questions as part of your response. Learning is often a matter of asking the right questions at the right time.

Definition 1

- ☐ Do points exist? In what sense do they exist or not?
- ☐ What does it mean that a point has no part?
- ☐ Modern definitions call a point a "precise location or place." How is this definition different from Euclid's?
- ☐ How does our concept of a point change if we define it as a location and not merely as a partless thing?

Definition 2

- ☐ Most modern definitions call a line a two dimensional figure that forms the shortest distance between two points and continues infinitely in both directions. How does Euclid's definition differ?

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[18] Don't forget to take notes in your journal as you read. By now it should be a habit. If it isn't a habit yet, find some way of reminding yourself every day until it becomes one. Write your reminder on a sticky note and put it in this book, if that helps. Or you could write your reminder in permanent marker on your little brother's forehead, so you'll be sure to see it throughout the day. On second thought, just go with the sticky note.

Epicycle B

Introduction

You'll remember reading a section in *Mathematics for the Nonmathematician* (3-6) that discussed axioms. Since mathematics is built upon self-evident truths, it's no surprise that *The Elements* begins by setting some forth. Euclid presents ten axioms, five of which appear as postulates, and five of which appear as common notions. But the axioms themselves are insufficient to do geometry. The reason for this, of course, is that geometry requires a specialized vocabulary, and a vocabulary requires definitions. As you'll soon see, without definitions, Euclid would find it difficult to prove anything.

Read

- ☐ Reread the definitions, postulates, and common notions.

Respond

Consider the following questions carefully. In your journal, write your answers to each question legibly and in full sentences.

Definition 3

- ☐ What is an extremity?

Definition 4

- ☐ Most modern textbook definitions assume lines to be straight. Why does Euclid add this definition of straight lines? Why is Def. 3 insufficient by itself?

Definition 5

- ☐ Why doesn't Euclid define a surface as being "flat"?
- ☐ What earlier definition(s) does Def. 5 resemble or parallel?

Definition 6

- ☐ What earlier definition(s) does Def. 6 resemble or parallel?

Definition 7

- ☐ What earlier definition(s) does Def. 7 resemble or parallel?

Epicycle C

Introduction

Notice the precision and extreme care Euclid uses with his terms. He takes nothing for granted, but is careful to lay out everything as explicitly as possible. If he didn't do this, his geometry wouldn't have the absolute certainty we expect of mathematics. Just as a house is only as sturdy as its foundation, mathematics is only as certain as its axioms, so we need to be absolutely sure about each one. After all, if your reasoning doesn't rest upon sound principles, how can you be absolutely sure that your conclusions are true?

We're spending time on the definitions and axioms for several reasons. First, they're obviously important. Second, by carefully examining the definitions, postulates, and common notions, we can begin to develop the habits of precise thinking that *The Elements* will later require of us. Third, it's a great opportunity to learn how to ask insightful questions of the reading. Take the study questions for models; they are exactly the kinds of questions you should learn to ask of the text.

<u>Read</u>

- ☐ Reread the definitions, postulates, and common notions.

<u>Respond</u>

Consider the following questions carefully. In your journal, write your answer to each question legibly and in full sentences.

Definition 9

- ☐ Why is it important to distinguish between rectilineal angles and non-rectilineal angles?

Definition 10

- ☐ In modern geometry we almost always use degrees to describe angles. Why does Euclid define right angles by equal adjacent angles, rather than in terms of a measurement of degrees?

Definitions 11 & 12

- ☐ Draw figures to represent the angles being defined.

Definition 13

- ☐ What are the boundaries of lines? of surfaces?

Definition 14

- ☐ Are points figures?

Definition 15

- ☐ What does it mean that the circle is the figure *contained* by the line? Is the line part of the circle?

Epicycle D

Introduction

Euclid's 5th postulate, also known as the parallel postulate, has caused mathematicians headaches for centuries. It may seem intuitively true, but, as ancient mathematicians realized, it reads more like a theorem than like a postulate. Mathematicians have long felt that the 5th postulate needs to be proved rather than assumed.

One Hungarian mathematician, Wolfgang Bolyai (1775-1856), expressed his frustration with the 5th postulate in a letter to his son, mathematician Johann Bolyai: "It is unbelievable that this stubborn darkness, this eternal eclipse, this flaw in geometry, this eternal cloud on virgin truth can be endured." Knowing the psychological consequences of trying to prove the 5th postulate, he added this warning: "You must not attempt this approach to parallels. I know this way to its very end. I have traversed this bottomless night, which extinguished all light and joy of my life. I entreat you, leave the science of parallels alone."[19]

To this day, no one has been able to prove that the 5th postulate is true. This strangely suggests that perhaps we are wrong to believe the 5th postulate. But if we reject it, then much of Euclid's geometry collapses.

As far as we know, the 5th postulate can't be proven, but neither has it been disproven. If it's true, our intuition is confirmed. If it's false, strange things happen. The rejection of the 5th postulate led to the creation of an alternate and mind-bending (I'd even call it psychedelic) system of geometry known as "Non-Euclidean" that we'll discuss another year.

For now, however, we simply have to assume that the 5th postulate is true. Like so much in life, we have to take it on faith.

<u>Read</u>

- ☐ Reread the definitions, postulates, and common notions.

<u>Respond</u>

Consider the following questions carefully. In your journal, write your answer to each question legibly and in full sentences.

Definitions 16 & 17
- ☐ Draw figures to represent the terms being defined.

Definition 18
- ☐ Why is the center of a semicircle the same as that of a circle?

Definitions 19 - 23
- ☐ Draw and label figures to represent each term being defined.

Postulates
- ☐ What is a postulate? Find out, and write your answer.

Common Notions
- ☐ How are the common notions different from the postulates?

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[19] Quoted from the St. John's College, Annapolis *Non-Euclidean Geometry Senior Mathematics Tutorial* Manual, p6.

CYCLE 6

The Elements, Book I - Euclid
I.1 - I.6

After completing this cycle, students should be able to answer the following questions & perform the following constructions.

- Why is I.4 important?
- What is the relationship between I.5 and I.6?
- Construct I.1, I.2, and I.3 in full.

NB: From this point forward, you will need a compass and straightedge.[20] You cannot properly perform the constructions without these two tools.

[20] Any ruler will do for a straightedge.

Epicycle A

Introduction

As we've seen, Euclid begins *The Elements* by presenting definitions, postulates, and common notions. Next, Euclid will present a few constructions and a key theorem on the congruence of triangles. With that foundation laid, a world is opened for discovery. I can't help but compare this beginning to the opening of Genesis. God creates the world out of nothing in six days, and, on a much smaller scale, Euclid creates a world out of concepts in 4 propositions.

<u>Read</u>

- ☐ Proposition I[21]

    Read the proposition carefully. In your journal, construct the figure according to Euclid's directions. Be sure to use your compass and straightedge.

NB: There are 4 parts to each proposition. Propositions begin with an enunciation, customarily given in italics at the beginning. Next, "givens" are presented to establish the starting point of the proposition by telling the reader what he can begin with. After that come the directions for constructing the figure. Finally comes the proof—the most important part.

<u>Respond</u>

Construction A

On a given finite straight line to construct an isosceles triangle with its equal sides equal to the given line.

Given:

Let AB be the given finite straight line.

Let circle DBC be described with center A and radius AB. [Post. 3]

Let line AC be joined to any point C on circle DBC such that AC does not lie in a straight line with AB. [Post. 1]

Let points C and B be joined with a straight line.

I say that triangle ABC is an isosceles triangle with side AC equal to AB.

Construct the figure in your journal and write a proof in the style of Euclid to demonstrate that the figure is truly an isosceles triangle. (Hint: Use definitions.)

Bonus challenge: There's a flaw in the given construction in this exercise. Find it and create a way to correct it. Don't give up if you can't figure it out right away. Think about it for a few days.

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[21] Each proposition in Book I should be memorized. You need not memorize them word for word, but you should memorize the processes required to construct the figures, as well as the gist of the arguments.

Epicycle B

Introduction

While Proposition I (I.1) gives us the ability to draw equilateral triangles of any size, I.2 and I.3 give us the ability to draw lines of any length. Notice I.2 gives us the ability to produce a line of a given length, but I.3 gives us the ability to cut off a given length from a preexisting line.

You'll notice that I.1, I.2, and I.3 end with the phrase, "Being what it was required to do." In later constructions, this is rendered in the conventional Latin as *quod erat faciendum.* Now, you won't see the Latin anywhere because it's conveniently abbreviated Q.E.F.

If you flip through a few propositions, you will also notice the abbreviation Q.E.D. that stands for a related Latin phrase: *quod erat demonstrandum*, which means something like "Being that which was to have been demonstrated." What you need to know is that constructions—propositions requiring you to build a figure—end with Q.E.F. while theorems—propositions concerned primarily with proving a particular truth—end with Q.E.D. This makes it easy to spot the difference.

Read

- ☐ Reread Proposition 1 and go on to study Propositions 2 and 3.[22]

Respond

Proposition 2

In your journal, construct the figure for I.2 according to Euclid's directions. Be sure to use your compass and straightedge. It is very important you do not abbreviate or skip any steps. Be sure to draw out I.1 in full whenever it is used. If you do this correctly, your diagram should look more complicated than the one in your book. (The diagram in your copy of *The Elements* is a cleaned-up version of the proposition where none of the intermediate steps are shown.)

Proposition 3

In your journal, construct the figure for I.3 according to Euclid's directions. Be sure to use your compass and straightedge. It is very important you do not abbreviate or skip any steps. Be sure to draw out I.1 and I.2 in full whenever they are used. As before, your diagram should look far more complicated than the one in your book. You should be able to spot the diagrams for I.1 and I.2 within your diagram for I.3.

Tip: Draw the figures on the larger side to make the small intermediate steps easier to manage. If you draw your figure too small, your diagram can get very messy and disorienting. Be careful, however, not to draw the figure too large accidentally. You don't want your compass drawing circles that extend onto the table.

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[22] Remember to record in your journal any observations or questions that arise as you read. A thoughtful question often leads toward a deeper understanding.

Epicycle C

Introduction

Proposition 4 deeply unsettled me the first time I read it. Before I explain why, we ought to consider why I.4 is so important. I.4 is the first proposition that attempts to prove anything. I.1 - I.3 were constructions, but I.4 is a theorem. I.4 proves what is known as the angle-side-angle theorem. The theorem gives us a way to compare triangles and say whether they're equal or unequal. This is so useful that nearly every proposition after I.4 relies upon it in some way.

As you've already guessed, however, there's a problem. In order to prove I.4, Euclid has us "apply" one triangle to the other. Geometry is supposed to be abstract and ideal, free from the limitations and imperfections of the physical world. But on what grounds do we allow the physical act of application into the ideal world of geometry? By requiring us to apply a triangle to another, he seems to be bringing unexamined physical notions into the pure and abstract realm of geometry. Furthermore, Euclid hasn't given us a postulate that says we can prove coincidence through application.

Modern textbooks often deal with these issues by presenting I.4 as an axiom. However, to my mind it's suspiciously complex for an axiom.

So we again find ourselves in an awkward position. We can't reject I.4 altogether, because the rest of *The Elements* would collapse. But to continue we must either accept application as a valid form of proof, or be willing to take I.4 entirely on faith as an axiom. Don't get the wrong idea; no one doubts that I.4 is true. The question is how we *know* it to be true. In other words, how far should we trust logic versus experience, and why?

<u>Read</u>

- ☐ Proposition 4

<u>Respond</u>

Construction B

On a given finite straight line to construct a rhombus.
Given: Let AB be the given finite straight line.

In your journal, complete the construction and give the proof. Remember to label your figure and write your proof in complete sentences. If you do this correctly, your construction should look and read like one of Euclid's propositions.

Hints: You will need to use Prop. 1 twice. Use C.N. 1. Use Def. 20. Use Def. 22.

Study Question

Consider the following question carefully. In your journal, write your answer legibly and in full sentences.

- ☐ Proposition 4 is immensely important. In what ways is it significantly different from the propositions which precede it?

Epicycle D

Introduction

Propositions 5 and 6 go together as a pair; I.6 is the converse of I.5. Euclid proves a set of relationships one way, and then provides a second proposition to prove the same relationship in the other direction. This is important because we often naïvely assume that the converse of a true statement is also true (e.g. 2+3 and 3+2 both equal 5). I once saw a well-meaning church sign that contained a grievous logical (and theological) error of this sort. It said, "GOD IS LOVE AND LOVE IS GOD." Yes, God is love because His character and being generate love, define love, and perfectly express love. However, it's stupid (not to mention heretical) to say that love is God. After all, not all love is holy nor pure.

So rather than assume that the converse of I.5 is true, Euclid takes care to prove it. He does this through a *reductio ad absurdum* proof.

Reductio ad absurdum is a method of proof in which you assume the opposite of what you're trying to prove and show that all cases but the true one are impossible because they result in absurdities. Thus, a lawyer might argue for the innocence of his client by agreeing that his client *could* be guilty, but then show that *if* he's guilty, none of the facts make sense. "If Mr. Jones committed the murder, he must be a fine marksman, since the victim was shot from half a mile away. The prosecution will admit that this is quite a feat, especially since Mr. Jones happens to be blind..."

<u>Read</u>

- ☐ Propositions 5 - 6

NB: Don't let the opening sentence of I.5 confuse you. By saying "Let there be an isosceles triangle," Euclid is not implying that he's already shown you how to construct one. Instead, he is taking the isosceles triangle as given, which means that he's assuming the existence of the isosceles triangle. For your notes, however, you will need to draw the isosceles triangle. (You can draw it like you did in Construction A.)

<u>Respond</u>

In your journal, draw out the diagram for I.5, carefully following Euclid's directions for its construction.

Study Questions

Consider the following questions carefully. In your journal, write your answer to each question legibly and in full sentences.

- ☐ How is I.4 used in I.5? Is it possible to prove I.5 without using I.4?
- ☐ I.6 is the converse of I.5. Why does Euclid provide a converse proposition to I.5?
- ☐ Consider the argument employed by I.6. How is it different from the arguments employed by the previous propositions?

CYCLE 7

The Elements, Book I - Euclid
I.7 - I.15

After completing this cycle, students should be able to answer the following questions & perform the following constructions.

- What themes does Euclid present from I.1 to I.15?
- What is the relationship between I.4 and I.8?
- Construct I.9, I.10, I.11, and I.12 in full.